

PERMUTATIONS AND COMBINATIONS

With and Without Repetition

permutations & combinations

3-digit PIN example (000–999)

1									
000	001	002	003	004	005	006	007	008	009
010	011	012	013	014	015	016	017	018	019
020	021	022	023	024	025	026	027	028	029
030	031	032	033	034	035	036	037	038	039
040	041	042	043	044	045	046	047	048	049
050	051	052	053	054	055	056	057	058	059
060	061	062	063	064	065	066	067	068	069
070	071	072	073	074	075	076	077	078	079
080	081	082	083	084	085	086	087	088	089
090	091	092	093	094	095	096	097	098	099
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127	128	129
130	131	132	133	134	135	136	137	138	139
140	141	142	143	144	145	146	147	148	149
150	151	152	153	154	155	156	157	158	159
160	161	162	163	164	165	166	167	168	169
170	171	172	173	174	175	176	177	178	179
180	181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198	199
200	201	202	203	204	205	206	207	208	209
210	211	212	213	214	215	216	217	218	219
220	221	222	223	224	225	226	227	228	229
230	231	232	233	234	235	236	237	238	239
240	241	242	243	244	245	246	247	248	249
250	251	252	253	254	255	256	257	258	259
260	261	262	263	264	265	266	267	268	269
270	271	272	273	274	275	276	277	278	279
280	281	282	283	284	285	286	287	288	289
290	291	292	293	294	295	296	297	298	299
300	301	302	303	304	305	306	307	308	309
310	311	312	313	314	315	316	317	318	319
320	321	322	323	324	325	326	327	328	329
330	331	332	333	334	335	336	337	338	339
340	341	342	343	344	345	346	347	348	349
350	351	352	353	354	355	356	357	358	359
360	361	362	363	364	365	366	367	368	369
370	371	372	373	374	375	376	377	378	379
380	381	382	383	384	385	386	387	388	389
390	391	392	393	394	395	396	397	398	399
770	771	772	773	774	775	776	777	778	779
780	781	782	783	784	785	786	787	788	789
790	791	792	793	794	795	796	797	798	799

Friends on a bench



Ice cream cones with 5 flavors

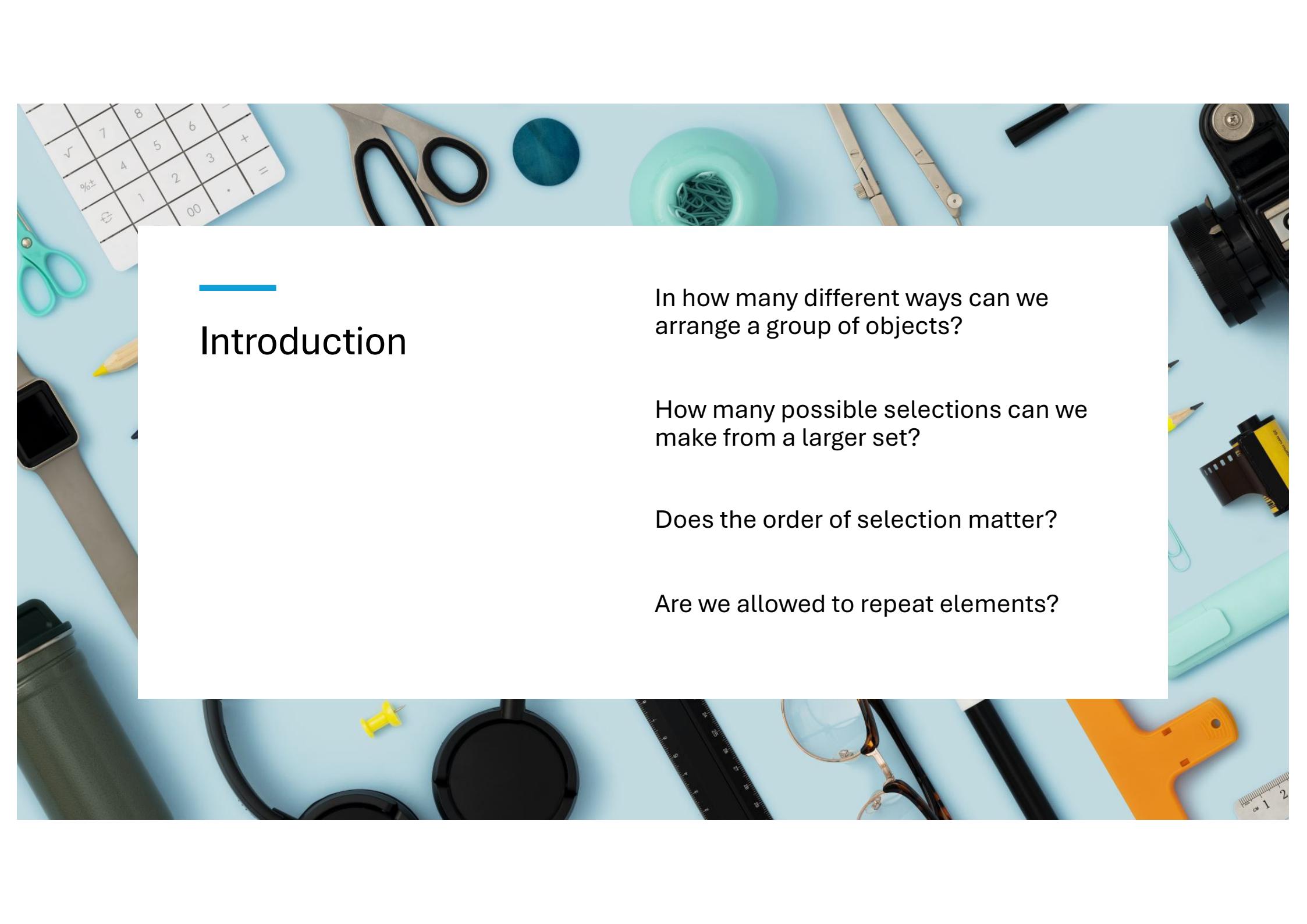


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Quick challenge!

How many ways can I arrange these 3 cards?





Introduction

In how many different ways can we arrange a group of objects?

How many possible selections can we make from a larger set?

Does the order of selection matter?

Are we allowed to repeat elements?

Introduction (contd.)



Permutations - arrangements where order matters



Combinations - selections where order does not matter

Motivation and relevance



Probability and Statistics



Computer Science and Data Science



Biology and Medicine



Economics and Social Sciences



Learning objectives

- Explain the difference between permutations and combinations
- Determine whether order matters in a given problem
- Compute permutations **without repetition**
- Compute permutations **with repetition**
- Compute combinations **without repetition**
- Compute combinations **with repetition**
- Apply the appropriate formulas to real-world problems

Fundamental counting principle (rule of product)

If one event can occur in **m** ways and a second independent event can occur in **n** ways, then the two events together can occur in

$m \times n$ ways.

Example

Suppose you have **3 shirts**, and **2 pairs of trousers**. The number of different outfits you can form is:

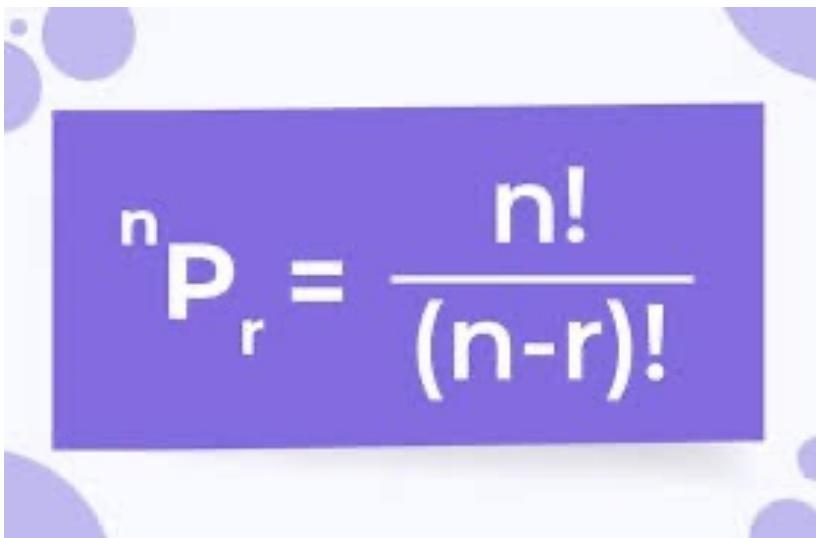
$$3 \times 2 = 6$$

This simple idea will now be extended to more complex counting problems.

Permutations (order matters)

A permutation is an arrangement of objects where the order is important.

For example, given a set {A, B, C}, the permutations ABC, ACB, BAC, BCA, CAB, and CBA are all distinct because they arrange the elements in different orders.


$${}^n P_r = \frac{n!}{(n-r)!}$$

Permutations without repetition

Problem Setup

Suppose we have **n distinct objects**, and we want to arrange **r** of them **without reusing any object**.

Derivation

- The first position can be filled in **n** ways
- The second position in **(n – 1)** ways
- The third position in **(n – 2)** ways
- Continue until **r** positions are filled

Formula

The number of permutations of **n objects taken r at a time** is:

$$P(n, r) = \frac{n!}{(n - r)!}$$

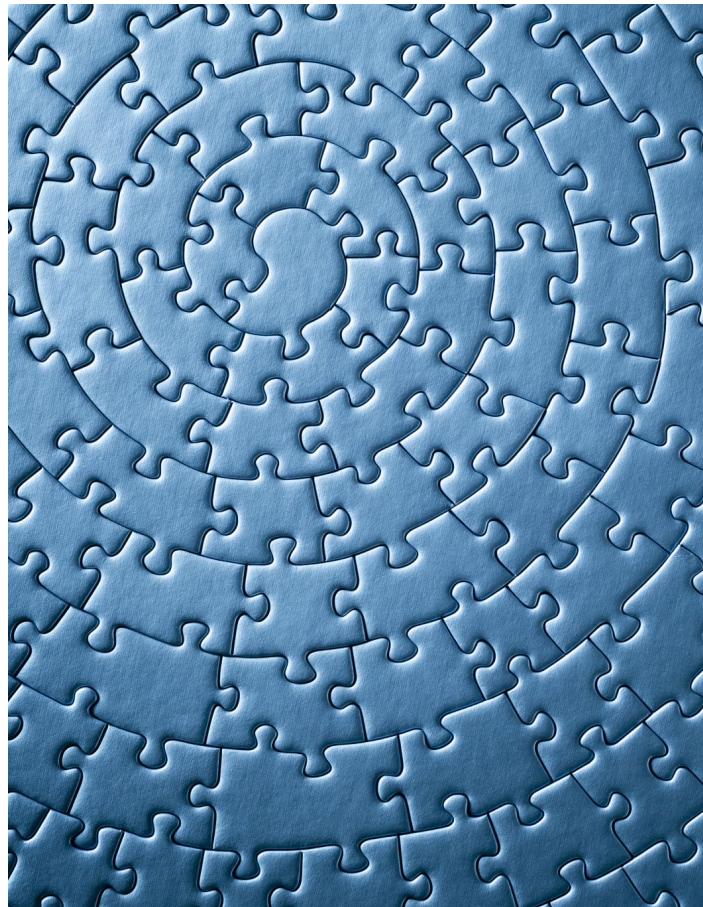
$$n! = n \times (n - 1) \times \cdots \times 2 \times 1$$

Example 1

In how many ways can 3 students be selected and arranged as student union representatives from 5 students?

Here: $n = 5$ and $r = 3$

$$P(5,3) = \frac{5!}{3!} = \frac{120}{2} = 60$$





Special case (permutations of all objects)

When **all n objects are arranged**, the number of permutations is:

$$P(n, n) = n!$$

Example 1

How many ways can 4 books be arranged on a shelf?

$$4! = 24$$

Example 2

In how many different ways can 4 students be seated in 4 distinct chairs arranged in a row?

Step 1: Identify the counting type

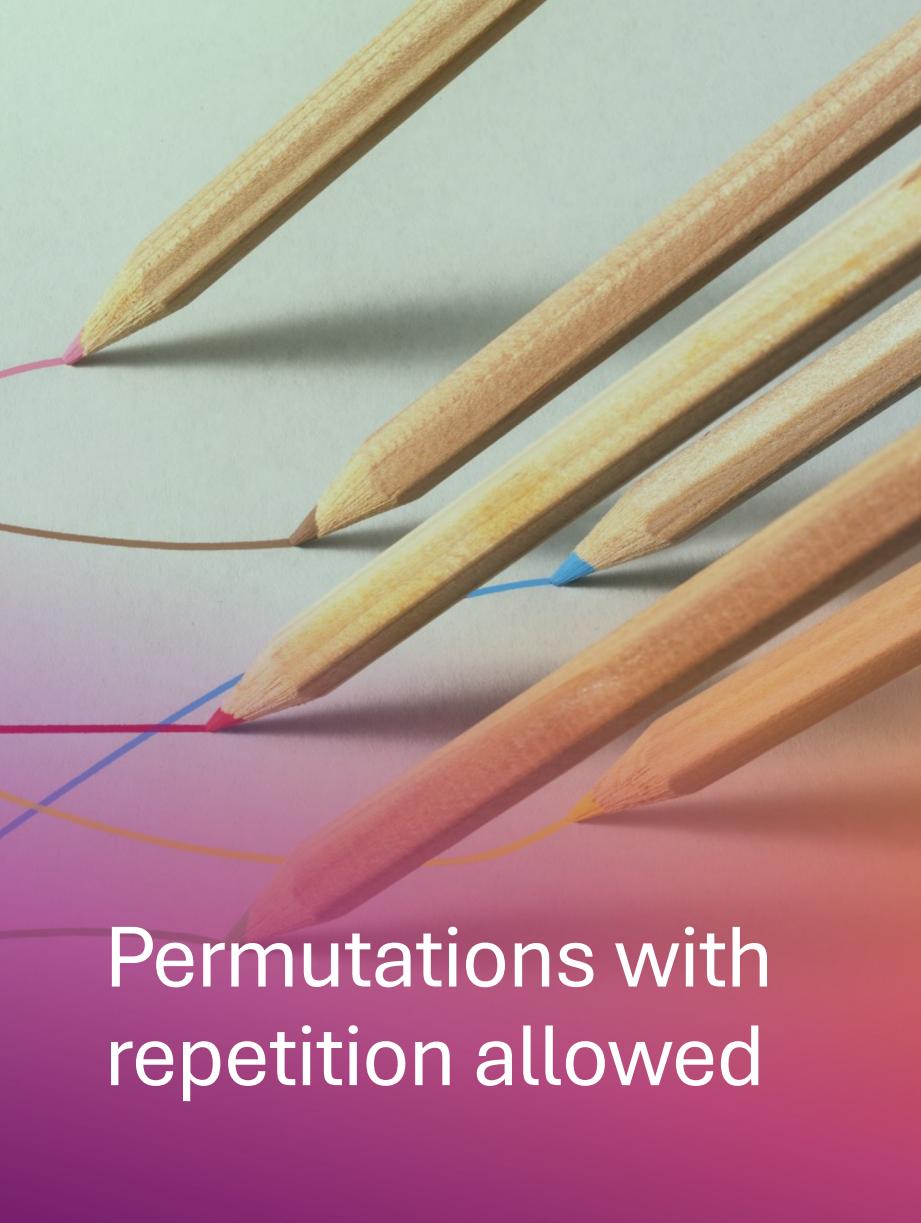
- **Order matters:** Sitting A–B–C–D is different from B–A–C–D.
- **No repetition:** Each student can occupy **only one chair**.

Therefore, this is a problem of **permutations without repetition**.

Step 2: Apply the formula

Here: $n = 4$ (number of students), $r = 4$ (number of positions)

$P(4, 4) = 4! = 24$ **different seating arrangements** of the 4 students.



Permutations with repetition allowed

Formula

If there are **n distinct objects** and we form an arrangement of **r positions**, allowing repetition, then the number of permutations is:

$$n^r$$

Example 1

How many **4-digit PIN codes** can be formed using the digits 0–9?

$$n = 10 \text{ digits}; r = 4 \text{ positions}$$
$$10^4 = 10,000$$

Here, repetition is allowed because digits can appear more than once.

Example 2

In how many different ways can a 3-letter password be formed using the letters $\{A, B, C, D\}$ if letters may be repeated?

Identify the Counting Type

- **Order matters:** ABC is different from BCA, so arrangement matters.
- **Repetition is allowed:** A letter can be used more than once (e.g., AAB, DDD).

Therefore, this is a case of **permutations with repetition**.

Finish up the solution

Combinations (order does not matter)

- A **combination** is a **selection of objects where order is not important**.
- If changing the order does **not** produce a new outcome, then we are dealing with combinations.

$\{A, B, C\}$ and $\{C, B, A\}$ are considered identical.

Combinations without repetition

Motivation

Consider selecting committee members:

- The group {A, B, C} is the same as {C, B, A}
- Order does not matter

Formula

The number of combinations of **n objects taken r at a time** is:

$$C(n, r) = \binom{n}{r} = \frac{n!}{r! (n - r)!}$$

Example 1

In how many ways can **3 students** be selected from **5 students** to form a committee?

$$C(5,3) = \frac{5!}{3! 2!} = \frac{120}{12} = 10$$



Example 2

A department has **8 students**. In how many different ways can a **research team of 3 students** be selected?

Identify the Counting Type

- **Order does not matter:** The team {A, B, C} is the same as {C, B, A}.
- **Repetition is not allowed:** A student cannot be selected more than once.

Therefore, this is a **combination without repetition** problem.

Identify Parameters

Number of students: $n = 8$; Number of students selected: $r = 3$

Finish up the solution

Combinations with repetition allowed

Situation

This occurs when:

- Objects can be chosen more than once
- Order does not matter

Formula

The number of combinations of **n distinct objects**, taken **r at a time**, with repetition allowed is:

$$C(n + r - 1, r)$$

Example 1

How many ways can **3 sweets** be selected from **5 types of sweets**, if repetition is allowed?

$$C(5 + 3 - 1, 3) = C(7, 3) = 35$$

Example 2

A cafeteria offers **5 types of fruits**: Apple, Banana, Orange, Mango, Pineapple. In how many different ways can a student select **3 fruits**, if the same type of fruit can be chosen more than once?

Identify the Counting Type

- **Order does not matter:** Selecting (Apple, Banana, Mango) is the same as (Mango, Banana, Apple).
- **Repetition is allowed:** The student may choose more than one fruit of the same type (e.g., Apple, Apple, Orange).

Therefore, this is a **combination with repetition** problem.

Finish up the solution





Common mistakes to avoid

- Using permutations when **order does not matter**
- Forgetting whether **repetition is allowed**
- Confusing $P(n, r)$ with $C(n, r)$

Summary table

Situation	Order Matters?	Repetition Allowed?	Formula
Permutation	Yes	No	$\frac{n!}{(n-r)!}$
Permutation	Yes	Yes	n^r
Combination	No	No	$\frac{n!}{r!(n-r)!}$
Combination	No	Yes	$C(n + r - 1, r)$