Discrete Structures
CSC 203

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Course Content

Part 1 (Dr. Afolabi)

Propositional Logic
Predicate Logic
Sets
Functions
Sequences and Summation
Proof Techniques. Mathematical induction

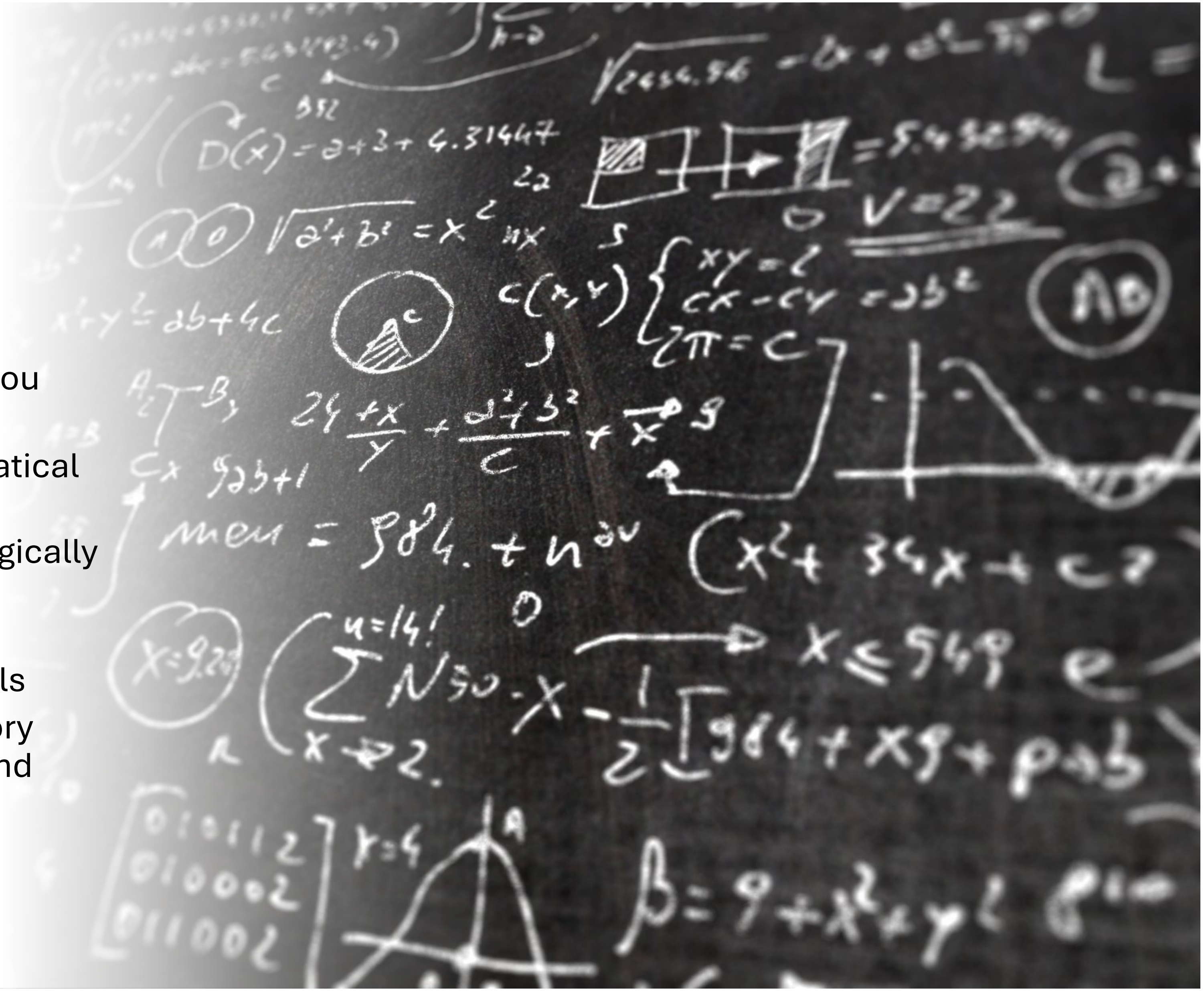
Part 2 (Dr. Nwohiri)

The principle of Inclusion-exclusion
The Pigeonhole principle.
Permutations and Combinations (with and without repetitions).
The Binomial Theorem.
Discrete Probability.
Recurrence Relations.

Skills you will develop

By the end of this course, you should be able to:

- Write clear mathematical arguments
- Analyze problems logically
- Translate real-world problems into mathematical models
- Understand the theory behind algorithms and data structures



Recommended texts

- Discrete Mathematics and Its Applications — Kenneth Rosen (comprehensive)
- Discrete Mathematics - An Open Introduction — Oscar Levin (free online)
- Doerr & Levasseur's Applied Discrete Structures — Al Doerr & Ken Levasseur (open access)
- Other specialized books covering graph theory or logic.

Principle of Inclusion-Exclusion (PIE)

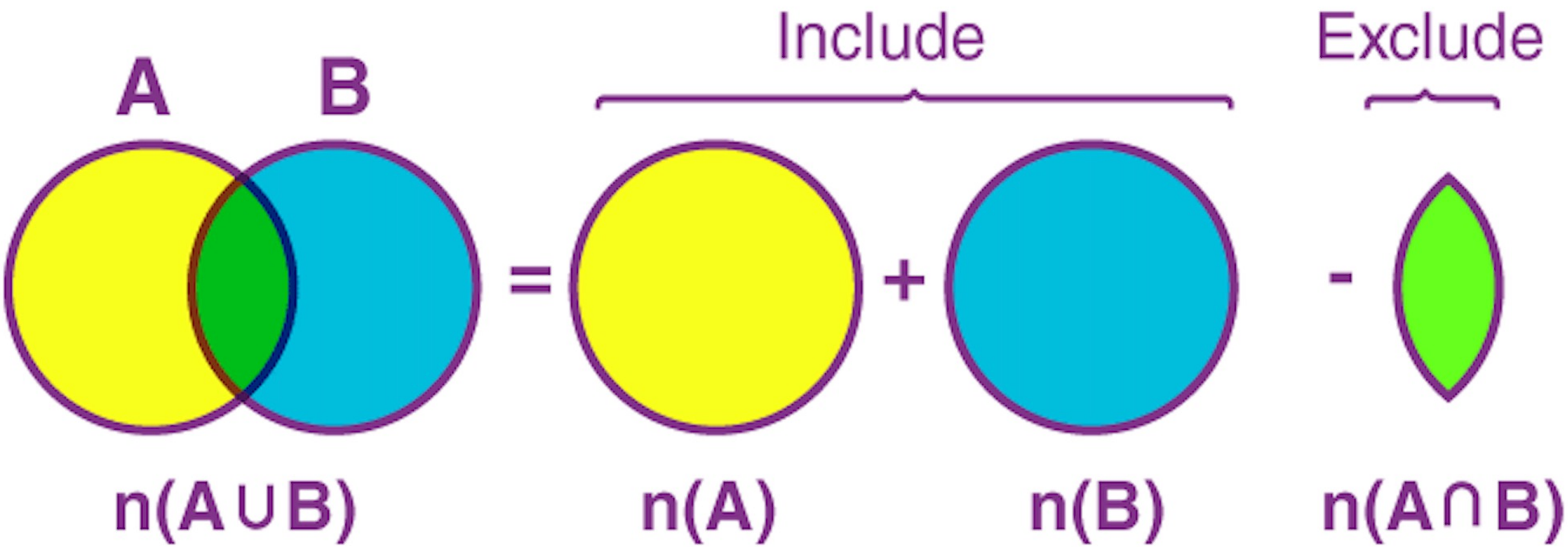
Motivation: Why Inclusion–Exclusion Is Needed

- In many counting and probability problems, we want to find the size of a **union of overlapping sets**.
- A common mistake is to **add everything** and forget about overlaps.

Simple question

If 120 students like tea and 90 like coffee, how many students like **tea or coffee**?

Principle of Inclusion-Exclusion (PIE)



A little about set theory

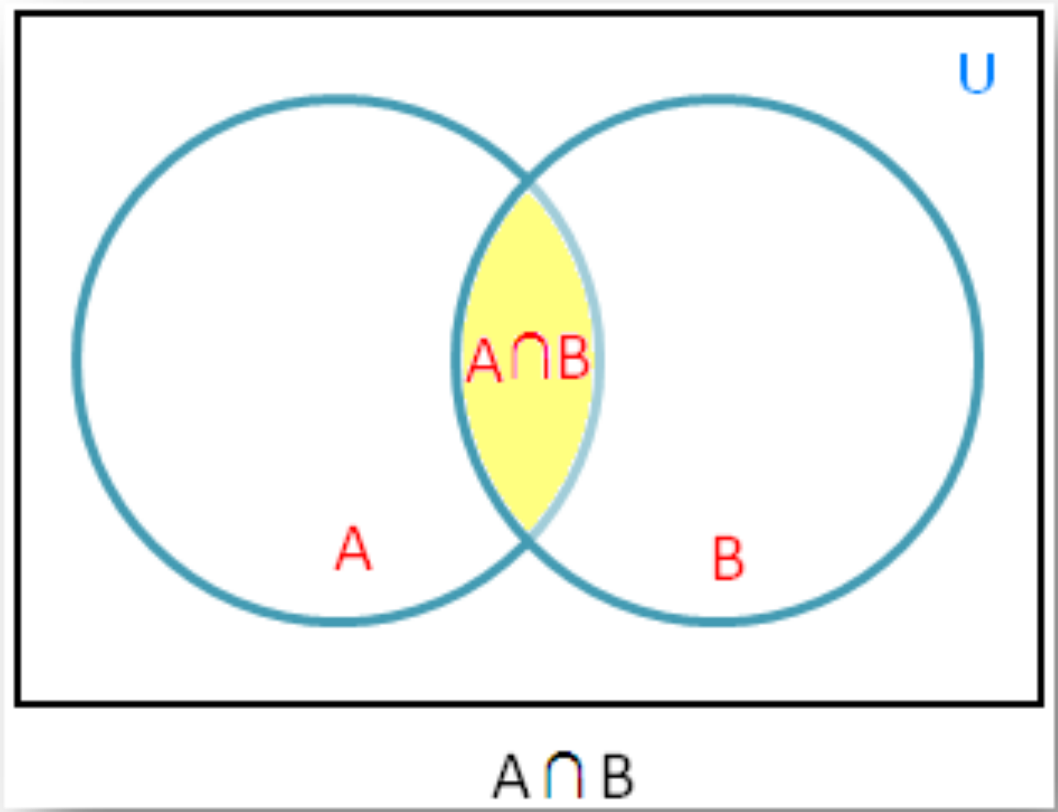
A ∪ B = the set of all elements that are in set **A**, or in set **B**, or in **both** sets.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Example

Consider two sets: **Set A** = {1, 2, 3, 4} and **Set B** = {3, 4, 5, 6}

A ∪ B = {1, 2, 3, 4, 5, 6}

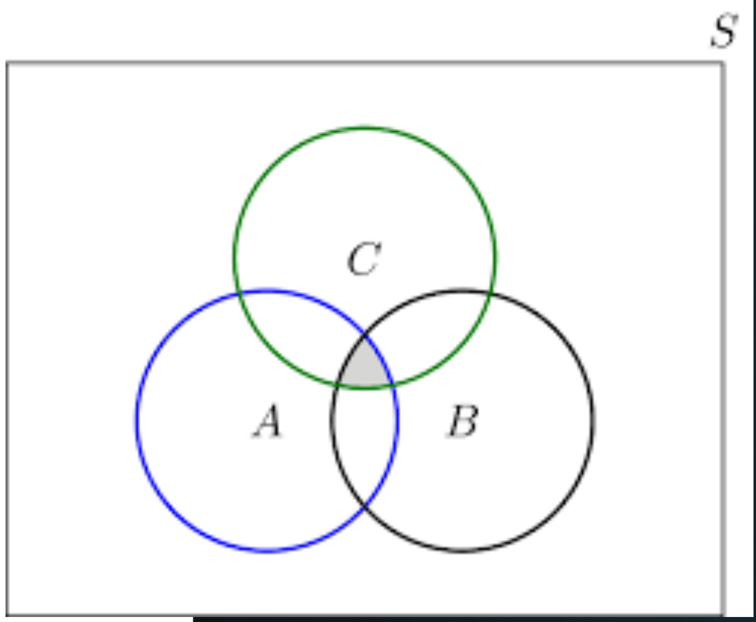


The basic counting problem

Suppose we have finite sets: $A,\ B,\ C$

We want to find: $|\ A \cup B \cup C\ |$

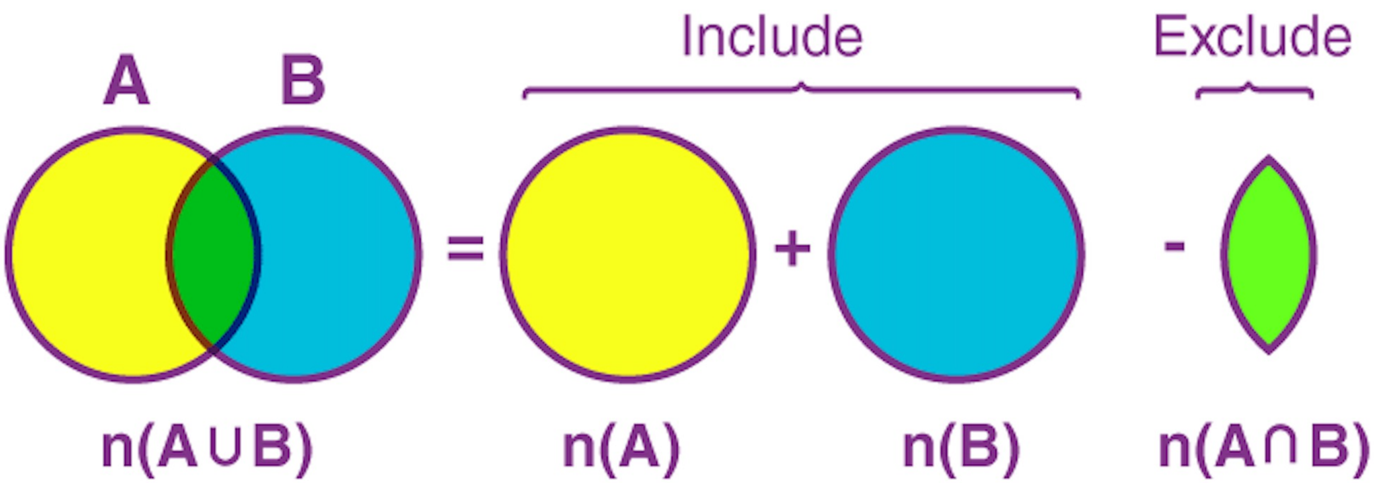
That is, **how many elements are in at least one of the sets**.



Two sets case

For two sets A and B:

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Example

In a class:

40 students take Mathematics

30 students take Physics

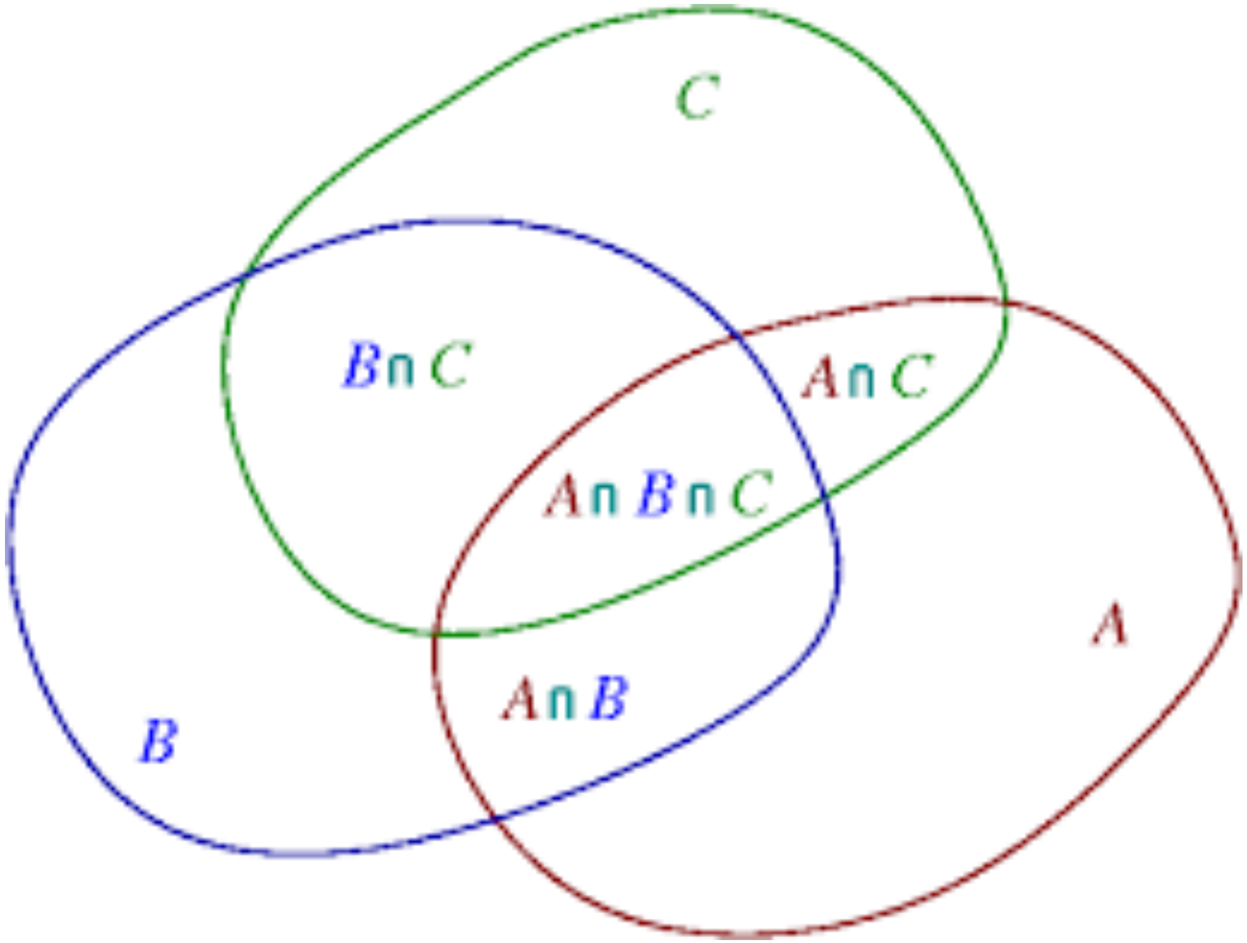
15 students take both

How many take at least one subject?

Three sets case

For three sets $A,\ B,\ C$:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



General Principle (n Sets)

For sets $A_1, A_2, \ldots, A_n$:

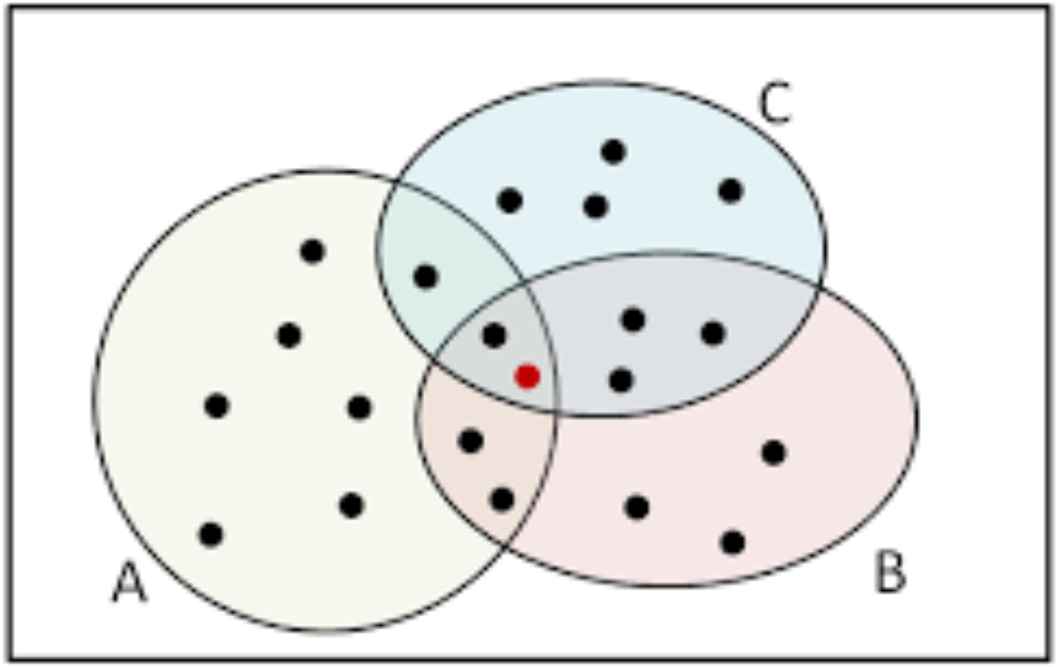
$$|\bigcup_{i=1}^{n} A_i| = \sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap A_2 \cap \cdots \cap A_n|$$

Venn diagram interpretation

PIE corresponds exactly to **correctly counting each region in a Venn diagram once**.

- Elements in exactly one set → counted once
- Elements in exactly two sets → added twice, subtracted once
- Elements in all three sets → added three times, subtracted three times, added once

Final count = 1



Applications - probability

Used to compute:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Applications - number theory

Counting integers divisible by given numbers.

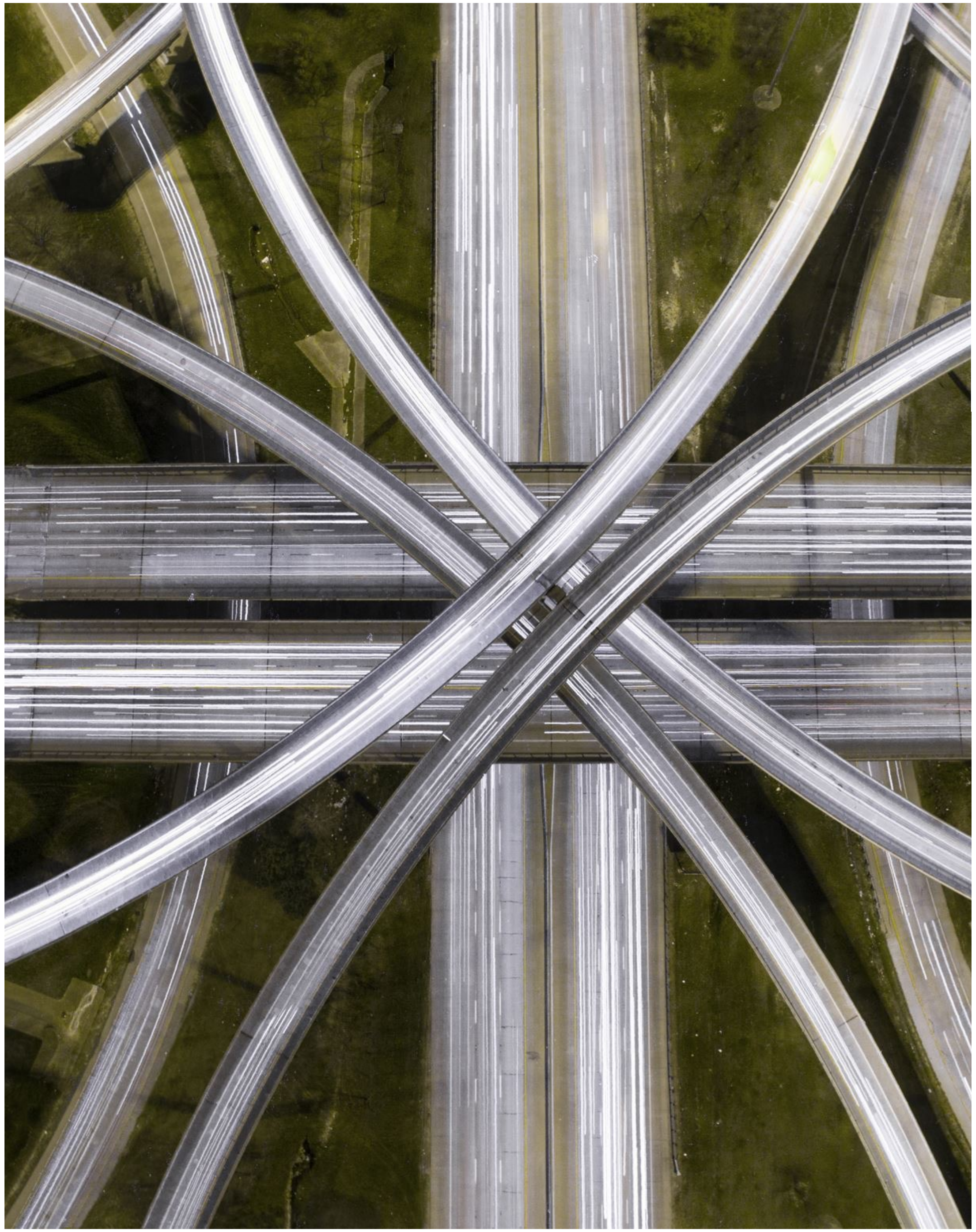
Example: Numbers from 1 to 100 divisible by 2 or 3:

- Divisible by 2: 50
- Divisible by 3: 33
- Divisible by 6: 16

$50 + 33 - 16 = 67$

Applications - Computer Science

- Counting valid passwords
- Analyzing constraints in algorithms
- Counting strings with forbidden patterns



Common mistakes

1. Forgetting to subtract overlaps
2. Forgetting to add back higher-order intersections
3. Stopping at pairwise terms when more sets exist

Practice problems

- In a survey: 120 students like tea, 90 like coffee, 50 like juice, 40 like tea and coffee, 30 like tea and juice, 20 like coffee and juice, 10 like all three. How many like **at least one drink**?
- How many integers from 1 to 1000 are divisible by 3, 5, or 7?
- What is the probability that at least one bit is 1 in a random 10-bit string?
- How many integers from 1 to 10,000 are divisible by 2, 3, or 5?
- A network has 5 independent failure points. Express system failure probability using PIE.

Key takeaways

- Inclusion–Exclusion corrects **overcounting**
- Add what you want, subtract overlaps, add back higher overlaps
- Signs alternate: $+-+-\cdots$
- Widely used in mathematics, statistics, probability, and computing