OSUN STATE UNIVERSITY

COLLEGE OF SCIENCE, ENGINEERING AND TECHNOLOGY

FACULTY OF BASIC AND APPLIED SCIENCES

DEPARTMENT OF PHYSICS

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COURSE CODE: PHY 101 COURSE: GENERAL PHYSICS 1 UNIT: 3

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WORK

- **Work *W* is the energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work. It is a scalar quantity**

- **The work done by a force can be defined as the product of the magnitude of the displacement and the component of the force in the direction of the displacement. The unit of work in S.I. is the Joule (J). 1 J = 1 N.m other unit of work is foot-pound (ft.lb), although old use.**

Mathematical expression of Work done

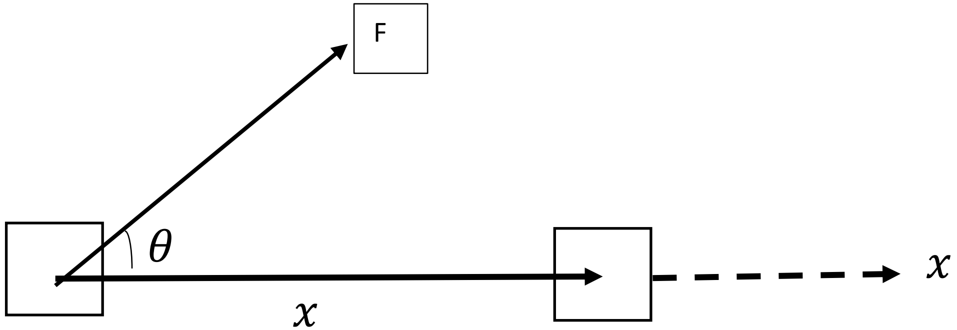


Fig. 1

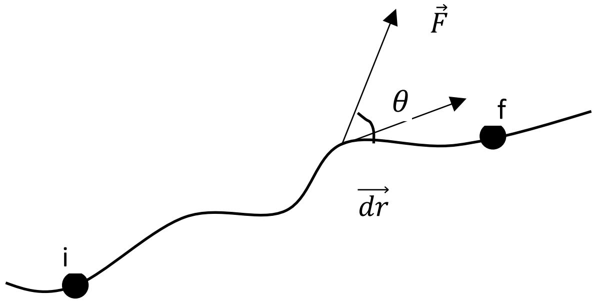


Fig. 2

Work can be expressed mathematically as

$$W = Fx\ cos\theta \qquad \ldots \qquad (1)$$

$$W = \vec{F}.\vec{x} \qquad \ldots \qquad (2)$$

Equation (2) is especially useful for calculating the work when $\vec{F}\ and\ \vec{x}$ are given in unit vector notations.

If a force displaces the particle through a distance *dr*, then the work done *dW* by the forced is

$$dW = \vec{F}.d\vec{r} \qquad \ldots (3)$$

The total work done by the force in moving the particle from initial point *I* to final point *f* will be

$$W = \int_i^f \vec{F}.d\vec{r} = \int_i^f F dr \cos\theta \qquad \ldots \qquad (4)$$

WORK-KINETIC ENERGY THEOREM

The work-kinetic energy theorem states that: **“when a body is acted upon by a force or resultant force, the work done by the force is equal to the change in kinetic energy of the body”.**

Proof: $W = \int_i^f \vec{F}\,.\,d\vec{r} = \int_i^f F dr \cos\theta$

$Force\ \vec{F}\ is\ in\ the\ direction\ of\ displacement\ d\vec{r}, \therefore\ \theta = 0$

$$W = \int_i^f F dr = \int_l^f madr$$

$$W = \int_l^f m\left(\frac{dv}{dt}\right)dr = \int_l^f m\left(\frac{dv}{dr}\frac{dr}{dt}\right)dr$$

$$W = \int_l^f m\frac{dv}{dr}vdr = \int_l^f mvdv = m\int_l^f vdv$$

$$W = m\left[\frac{v^2}{2}\right]_l^f = m\left[\frac{v_f^2}{2} - \frac{v_i^2}{2}\right]$$

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \ \ldots \qquad (5)$$

$$W = k_f - k_i$$

$$W = \Delta K \qquad \ldots \qquad (6)$$

POWER

Power is the time rate at which work is done. If an amount of work ΔW is done in a small interval of time Δt. It is the scalar product of $\vec{F}$ *and* $\vec{v}$. The S.I. unit of power is J/S, which 1 Watt. (1 W = 1 J/S). Power P is expressed as

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad (7)$$

$$P = \frac{\Delta W}{\Delta t} = \vec{F}.\frac{dW}{dt} = \vec{F}.\vec{v} \qquad (8)$$

Recall that instantaneous velocity

$$V = \frac{dr}{dt}$$

If $\vec{F}$ acts at an angle of θ to v, then

$$P = \vec{F}.\vec{v} = Fv\cos\theta \ \ldots\ldots\ldots \qquad (9)$$

POTENTIAL ENERGY (U), POSITIVE, NEGATIVE AND ZERO WORK

- Potential energy U is associated with system of bodies or objects under the action of forces.
- Assuming a woman who carries a load on her head is static. She does no work so long she makes no displacement, although she may be sweating and tired!
- This is because the component of the force on her is vertical, i.e $\theta = 0$ and Cos$\theta = 0$.
- The work done on the load by her neck/head is negative. You can remember Newton's third law of motion supporting this.

$$\Delta U = -W$$

Remember that

$$W = \int_i^f F dr$$

$$\therefore \ \Delta U = -\int_i^f F dr$$

POTENTIAL ENERGY (U), POSITIVE, NEGATIVE AND ZERO WORK

If gravitational, it is the potential energy associated with objects under the influence of gravitational force, i.e a system of Earth and nearby particle.

$$\Delta U = -\int_i^f F dr$$

$$= -\int_i^f (-mg) dr$$

$$= mg \int_i^f dr \;\; mg\,(r)\Big|_i^f$$

$$\Delta U = mg(r_f - r_i)$$

Thus, taking the starting point to be zero, then

$$\Delta U = mgr$$

ELASTIC POTENTIAL ENERGY

- Elastic potential energy is the energy stored up in elastic materials.
- It is associated with the state of the object whether compressed or stressed.
- For a spring which exerts a force $f = -kx$

$$\Delta U = -\int_{x_i}^{x_f} F dx$$

$$= -\int_{x_i}^{x_f} (-kx) dx$$

$$= k\int_{x_i}^{x_f} (x) dx$$

$$= k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$$

$$= \frac{1}{2}k{x_f}^2 - \frac{1}{2}k{x_i}^2$$

WORKED EXAMPLES

Example 1:

A boy pushes a box with a force of 120 N at an angle of 30^0 to the horizontal, through a distance of 10 m. how much work is done by the boy?

Solution: $W = Fx\ cos\theta$

$W = (120\ N)(10\ m)\ cos\ 30^0$

$W = 1.039 \times 10^3\ J$

Example 2:

A girl pulls her luggage with a constant force $\vec{F} = (80\ N)\hat{\imath} - (40\ N)\hat{\jmath}$ through a displacement of $\vec{x} = (10\ m)\hat{\imath} + (12\ m)\hat{\jmath}$ How much work is done by the girl on her luggage?

Solution:

Recall that $P = \overrightarrow{W} = \vec{F}.\vec{s} = F_x x + F_y y + F_z z$

$= (80\ N)(10\ m) + (-40\ N)(12\ m)$

$= 3.2 \times 10^3\ J$

CLASS WORK

Class work 1:

A student representative, drags a public address (PA) system weighing 7000 N through a distance of 20 m to the PHY 101 lecture theater. The student rep exerts a force of 3000 N at an angle of 40^0 to the horizontal. If a 2000 N frictional force opposes the motion of the public address system, find the work done by each force acting on the PA and the total work done by all the forces.

Class work 2:

A particle is acted upon by the forces

$$F_1 = 5i - 10j + 15k$$

$$F_2 = 10i - 25j - 20k, and$$

$$F_3 = 15i - 20j + 10k$$

Find the force needed to keep the particles in equilibrium.

SOLUTION TO CLASS WORKS

Class work 1 solution

Work done by the weight is zero, Work done by normal force is also zero

It remains work done by the force exerted by the student rep and the work done by the friction.

Thus, the total active force acting on the PA is

$F_T = F\ cos\theta$ + (- Frictional force)

F_T = 3000 N cos 40^0 - 2000 N

F_T = 298.13 N

The total work done is $W = \sum F_x\ x\ cos\theta$ = (298.13 N)(20 m) = 5962.6 J

Class work 2 solution

The resultant force is R

R $= F_1 + F_2 + F_3 = (5i - 10j + 15k) + (10i + 25j - 20k) + (15i - 20j + 5k)$

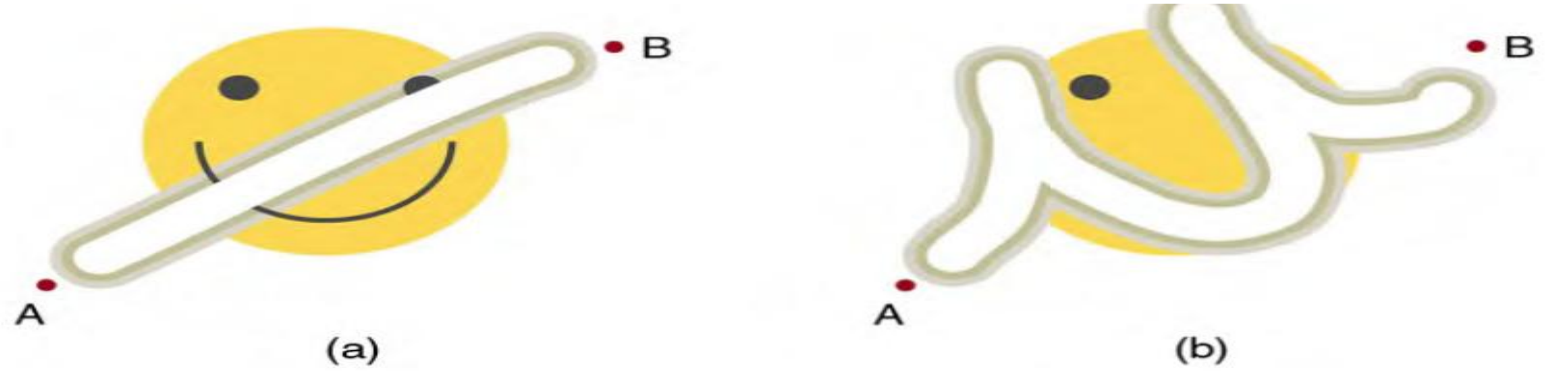
$= 30i - 5j + 5k$

The force to keep this in equilibrium

$= -R = -30i + 5j - 5k$

Nonconservative Forces

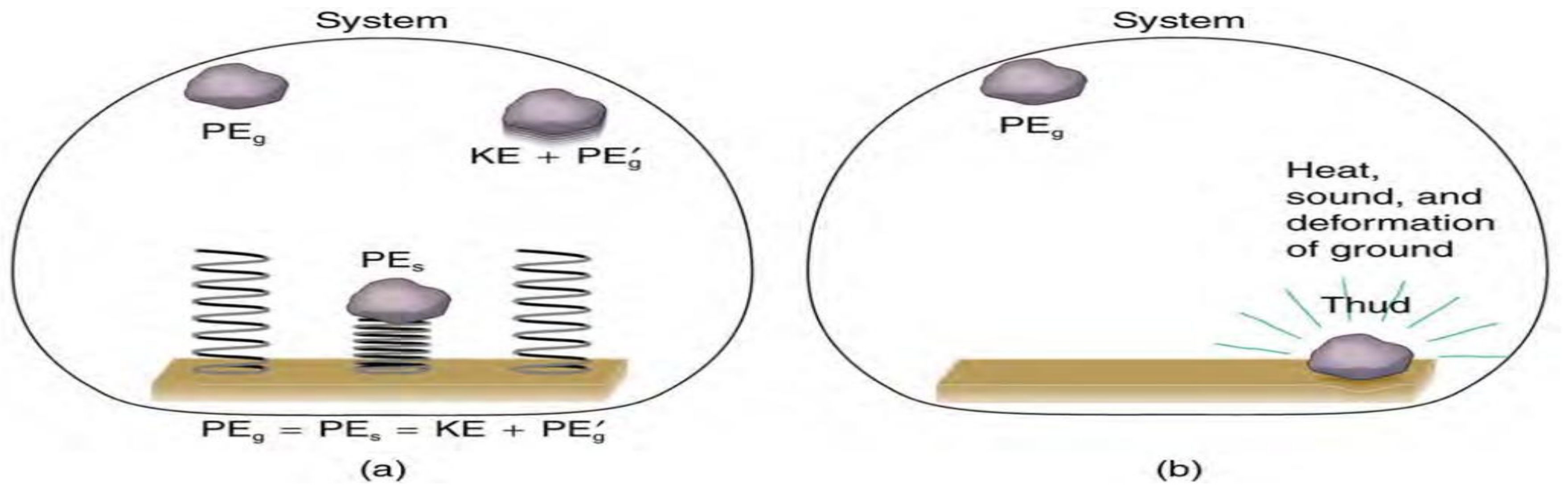
- A **nonconservative force** is one for which work depends on the path taken.
- Friction is a good example of a nonconservative force.
- An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*.
- **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system.



The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

How Nonconservative Forces Affect Mechanical Energy

- *Mechanical* energy *may* not be conserved when nonconservative forces act.
- For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy.



Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

Linear Momentum and Force

Linear momentum is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as $\mathbf{p} = m\mathbf{v}$.
The SI unit for momentum is kg · m/s

Example: (a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hardthrown 0.410-kg football that has a speed of 25.0 m/s.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation. $\mathbf{p} = m\mathbf{v}$

$$p_{\text{player}} = (110\ \text{kg})(8.00\ \text{m/s}) = 880\ \text{kg} \cdot \text{m/s}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$\mathbf{p} = m\mathbf{v}$

$$p_{\text{ball}} = (0.410\ \text{kg})(25.0\ \text{m/s}) = 10.3\ \text{kg} \cdot \text{m/s}$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

Newton stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$\mathbf{F}_{net} = \Delta \mathbf{p}/\Delta t$,

where $\mathbf{F}_{net}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

Linear Momentum and Force

Change in momentum $\Delta \mathbf{p}$ is given by $\Delta \mathbf{p} = \Delta(m\mathbf{v})$.

If the mass of the system is constant, then $\Delta(m\mathbf{v}) = m\Delta \mathbf{v}$.

So that for constant mass, Newton's second law of motion becomes $\mathbf{F}_{net} = \Delta \mathbf{p}/\Delta t = m\Delta \mathbf{v}/\Delta t$.

Because $\Delta \mathbf{v}/\Delta t = \mathbf{a}$, ∴ $\mathbf{F}_{net}=m\mathbf{a}$

WORKED EXAMPLES

Example 1: During the in the US Open at USTA Billie Jean King National Tennis Center on September 8, 2022, Ons Jabeur of Tunisia hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Ons Jabeur's racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Solution

When mass is constant, the change in momentum is given by $\Delta p = m\Delta v = m(v_f - v_i)$.

To determine the change in momentum, substitute the values for the initial and final velocities into the equation,

$$\Delta p = m(v_f - v_i)$$

$$= (0.057\ \text{kg})(58\ \text{m/s} - 0\ \text{m/s})$$

$$= 3.306\ \text{kg} \cdot \text{m/s} \approx 3.3\ \text{kg} \cdot \text{m/s}$$

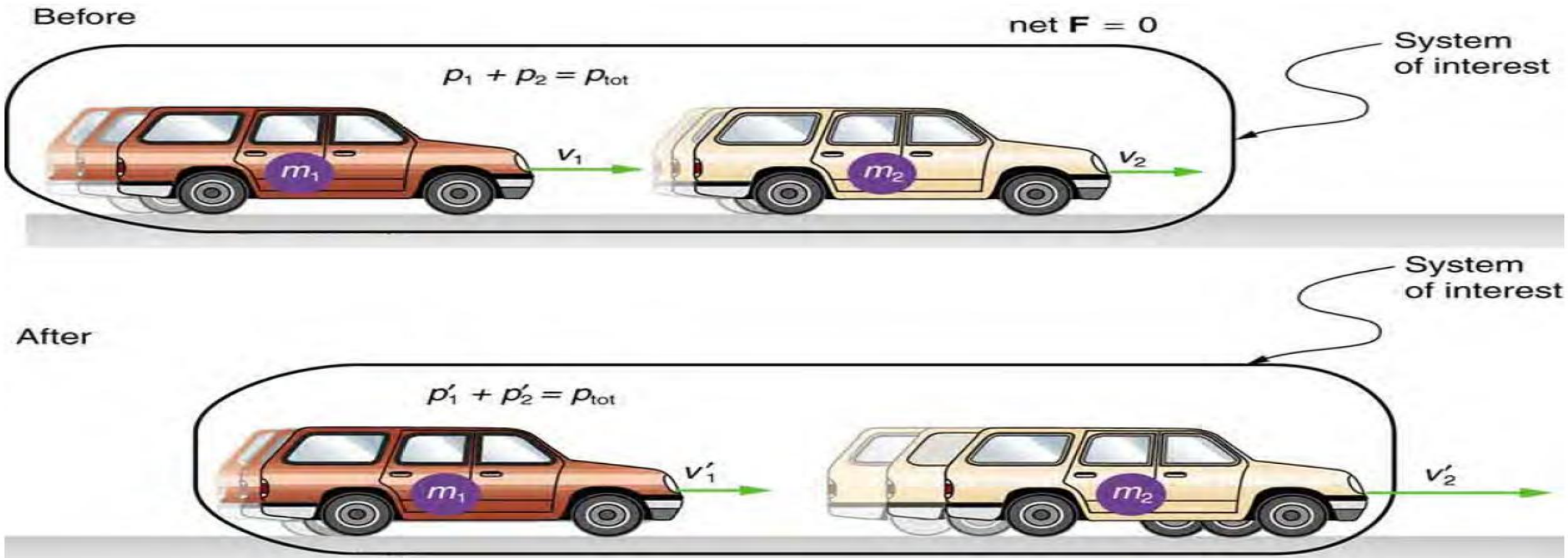
Now the magnitude of the net external force can determined by using $F_{net} = \Delta p/\Delta t$:

$$F_{net} = \Delta p/\Delta t = 3.306\ (\text{kg}\cdot\text{m/s})/5.0\times10^{-3}\ \text{s}$$

$$= 661\ \text{N} \approx 660\ \text{N}$$

Conservation of Momentum

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another



A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of v′1 and the second speeds up to a velocity of v′2 . The momentum of each car is changed, but the total momentum *p*tot of the two cars is the same before and after the collision (if you assume friction is negligible).

Change in Momentum

$\Delta p_1 = F_1 \Delta t$,
$\Delta p_2 = F_2 \Delta t$
where F_2 is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars.
We know from Newton's third law that $F_2 = -F_1$,
and so $\Delta p_2 = -F_1 \Delta t = -\Delta p_1$
Thus, the changes in momentum are equal and opposite, and $\Delta p_1 + \Delta p_2 = 0$.
Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,
$p_1 + p_2 = \text{constant}$,
$p_1 + p_2 = p'_1 + p'_2$,

where p'_1 and p'_2 are the momenta of cars 1 and 2 after the collision.
In equation form, the **conservation of momentum principle** for an isolated system is written
$\mathbf{p}_{tot} = \text{constant}$,
Or $\mathbf{p}_{tot} = \mathbf{p}'_{tot}$,
where $\mathbf{p}_{tot}$ is the total momentum and $\mathbf{p}'_{tot}$ is the total momentum some time later. An **isolated system** is defined to be one for which the net external force is zero $\mathbf{F}_{net} = 0$.

Elastic Collision

An **elastic collision** is one that conserves internal kinetic energy.

CONSERVATION OF MOMENTUM

When a collision occurs in an isolated system, the total momentum of the system doesn't change with the passage of time. Instead, it remains constant both in magnitude and in direction. The momenta of the individual objects in the system may change, but the vector sum of *all* the momenta will not change. The total momentum, therefore, is said to be *conserved*.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

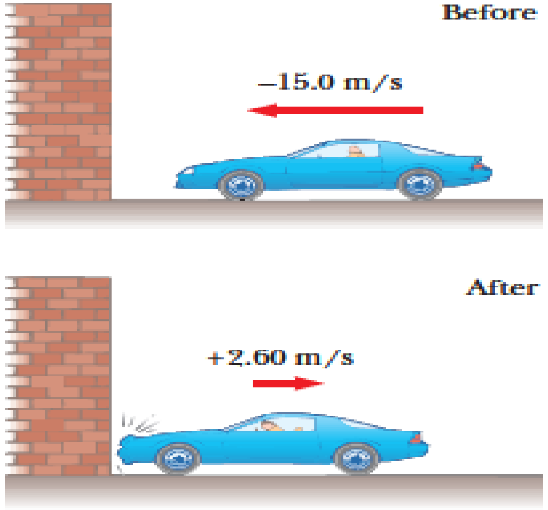
When no net external force acts on a system, the total momentum of the system remains constant in time.

WORKED EXAMPLE

A 0.150-kg baseball, thrown with a speed of 40.0 m/s, is hit straight back at the pitcher with a speed of 50.0 m/s.

(a) What is the impulse delivered by the bat to the baseball?

(b) (b) Find the magnitude of the average force exerted by the bat on the ball if the two are in contact for 2.00 10_3 s.



Solution (a) Find the impulse delivered to the car.

Calculate the initial and final momenta of the car:

$$p_i = mv_i = (1.50 \times 10^3\ \text{kg})(-15.0\ \text{m/s})$$
$$= -2.25 \times 10^4\ \text{kg} \cdot \text{m/s}$$

$$p_f = mv_f = (1.50 \times 10^3\ \text{kg})(+2.60\ \text{m/s})$$
$$= +0.390 \times 10^4\ \text{kg} \cdot \text{m/s}$$

The impulse is just the difference between the final and initial momenta:

$$I = p_f - p_i$$
$$= +0.390 \times 10^4\ \text{kg} \cdot \text{m/s} - (-2.25 \times 10^4\ \text{kg} \cdot \text{m/s})$$
$$I = 2.64 \times 10^4\ \text{kg} \cdot \text{m/s}$$

(b) Find the average force exerted on the car.

Apply Equation 6.6, the impulse–momentum theorem:

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4\ \text{kg} \cdot \text{m/s}}{0.150\ \text{s}}$$

$$= +1.76 \times 10^5\ \text{N}$$

CLASS WORK

1. An archer stands at rest on frictionless ice and fires a 0.500-kg arrow horizontally at 50.0 m/s. (See Fig.6) The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

(Fig.6)

2. A 70.0-kg man and a 55.0-kg woman on ice skates stand facing each other. If the woman pushes the man backwards so that his final speed is 1.50 m/s, at what speed does she recoil?

3. A car and a large truck traveling at the same speed collide head-on and stick together. Which vehicle experiences the larger change in the magnitude of its momentum? (a) the car (b) the truck (c) the change in the magnitude of momentum is the same for both (d) impossible to determine

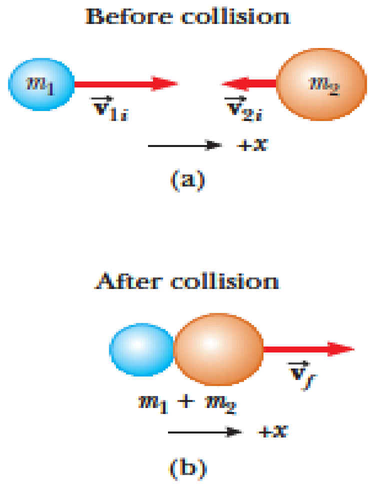


FIG. 7 (a) Before and (b) after a perfectly inelastic head-on collision between two objects.

CENTER OF MASS AND SYSTEMS OF PARTICLES

- *The center of mass of a body is defined as a point where the entire mass of the body appears to be concentrated.*
- Therefore, this point can represent the entire body.

- For bodies of regular shape and uniform mass distribution, the center of mass is at the geometric center of the body. Some examples are tabulated below:

Bodies of regular shape and uniform mass distribution	Geometric center of the body
circle and sphere	Centers of the circle and sphere
square and rectangle	Point of intersection of their diagonals
cube and cuboid	Point of intersection of their body diagonals

Center of Mass for Distributed Point Masses

- *A point mass is a hypothetical point particle which has nonzero mass and no size or shape.*
- To find the center of mass for a collection of n point masses, say, m_1, m_2, m_3 . . . m_n we have to first choose an origin and an appropriate coordinate system as shown in **Fig. C1**
- Let, x_1, x_2, x_3 . . . x_n be the X-coordinates of the positions of these point masses in the X direction from the origin.

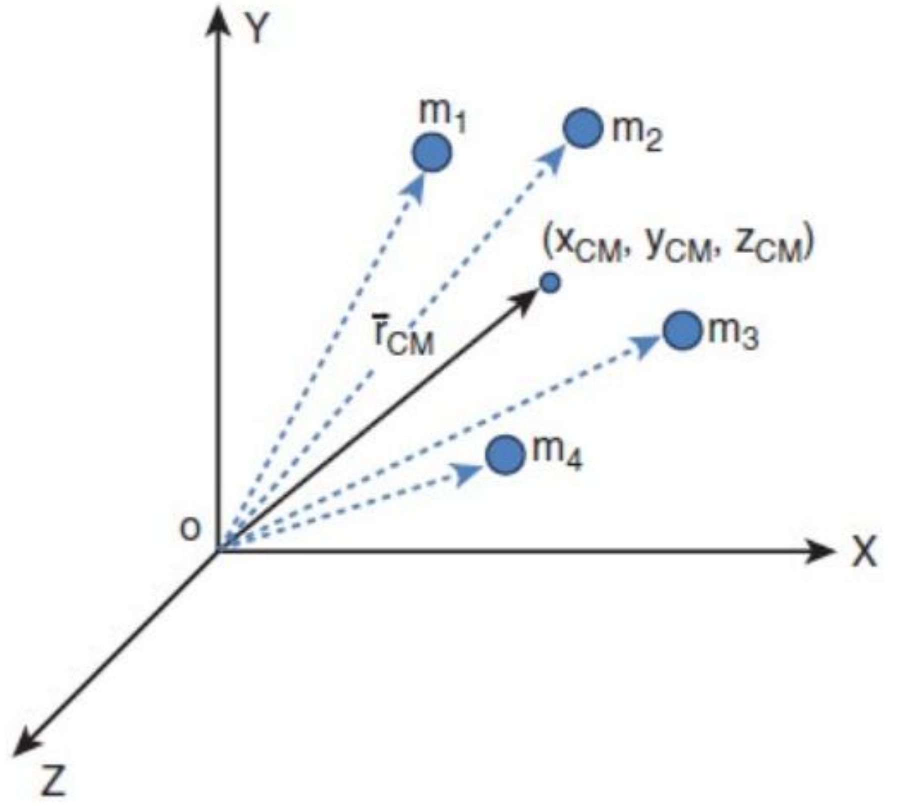


Fig. C1: center of mass for a collection of n point masses

Center of Mass for Distributed Point Masses Cont.

The equation for the X coordinate of the center of mass is,

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i}$$

where, $\sum m_i$ is the total mass M of all the particles, $\left(\sum m_i = M\right)$. Hence,

$$x_{CM} = \frac{\sum m_i x_i}{M}$$

$$y_{CM} = \frac{\sum m_i y_i}{M}$$

$$z_{CM} = \frac{\sum m_i z_i}{M}$$

Hence, the position of center of mass of these point masses in a Cartesian coordinate system is (x_{CM}, y_{CM}, z_{CM}).

Position of Center of Mass

Generally, the position of center of mass can be written in a vector form as,

$$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{M}$$

where, $\vec{r}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$ is the position vector of the center of mass and $\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$ is the position vector of the distributed point mass; where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along X, Y and Z-axes respectively.

Center of Mass of Two Point Masses

The center of mass of two point masses m_1 and m_2, which are at positions x_1 and x_2 respectively on the X-axis.

(i) When the masses are on positive X-axis:

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

(ii) When the origin coincides with any one of the masses:

$$x_{CM} = \frac{m_1(0) + m_2x_2}{m_1 + m_2}$$

$$x_{CM} = \frac{m_2x_2}{m_1 + m_2}$$

(iii) When the origin coincides with the center of mass itself:

$$0 = \frac{m_1(-x_1) + m_2x_2}{m_1 + m_2}$$

$$0 = m_1(-x_1) + m_2x_2$$

The equation is *principle of moments.*

$$m_1x_1 = m_2x_2$$

Ex 1. A uniform disc of mass 100g has a diameter of 10 cm. Calculate the total energy of the disc when rolling along a horizontal table with a velocity of 20 cms-1. (take the surface of table as reference)

Mass of the disc m = 100 g = 0.1 kg.
Diameter of the disc d = 10 cm
Radius of the disc r = 5 cm = 0.05m
Rolling with a velocity v = 20 cms_{-1} = 0.20 ms_{-1}
Total energy of the disc E_{Tot} = ?
E_{Tot} = Translational K.E. + rotational K.E.
Moment of inertia (M.I) of the disc about its own axis
:. I = 1.028 units

Moment of inertia (M.I) of the disc about its own axis

$$I = \frac{1}{2} mr^2 ; \; v = r\omega \; \therefore \; \omega^2 = \frac{v^2}{r^2} .$$

$$\text{Rotational K.E.} = \frac{1}{2} I\omega^2 = \frac{1}{2} \times \left(\frac{1}{2} mr^2\right) \times \left(\frac{v^2}{r^2}\right)$$

$$= \frac{1}{4} mv^2 .$$

$$\text{T.E.} = \frac{1}{2} mv^2 + \frac{1}{4} mv^2 = \frac{3}{4} mv^2$$

$$\text{T.E. of the disc } E_{Tot} = \frac{3}{4} \times 0.1 \times 0.20 \times 0.20$$

$$= 0.003 \text{ J.}$$

Ex 2. A particle of mass 5 units is moving with a uniform speed of $v = 3\sqrt{2}$ units in the XOY plane along the line y = x + 4. Find the magnitude of angular momentum.

Equation of line $x - y + 4 = 0$

Mass of particle = 5 units

Speed v - 3 √2 units

Distance of line from origin r =

$$r = \frac{0-0+4}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}}$$

.

Angel momentum L = mvr = 5 × 3 √2 × 4/√2 = 60 units

Ans: 60 units

Ex 3. **A fly wheel rotates with a uniform angular acceleration. If its angular velocity increases from 20π rad/s to 40π rad/s in 10 seconds. Find the number of rotations in that period.**

Initial Angular Velocity $\omega_0 = 20\pi$ rad s^{-1}
Final Angular velocity $\omega = 40\pi$ rad s^{-1}
time taken, t = 10 s
No. of rotations / s = ?

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{40\pi - 20\pi}{10} = \frac{20\pi}{10} = 2\pi$$

$$\alpha = 2 \text{ rad s}^{-1}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 20\pi \times 10 + \frac{1}{2} \times 2\pi \times 10 \times 10$$

$$= 200\pi + 100\pi$$

$$\theta = 300\pi.$$

$$\text{No. of rotations / sec.} = \frac{\theta}{2\pi} = \frac{300\pi}{2\pi}$$

$$= 150 \text{ rotations.}$$

Solved Problems

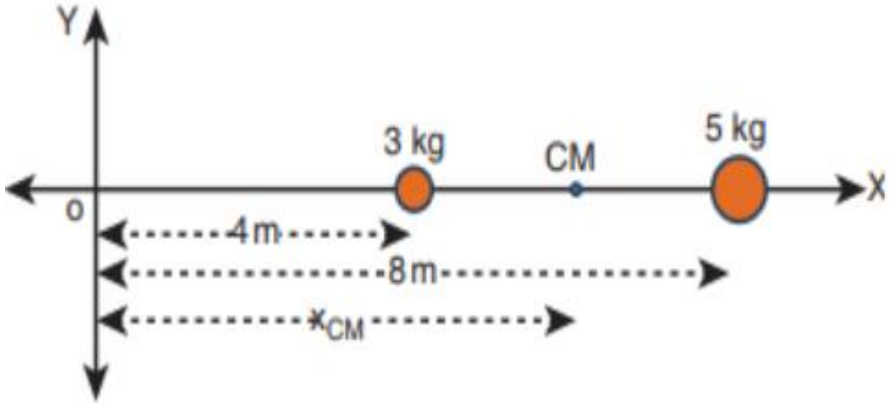
Ex.1. Two point masses 3 kg and 5 kg are at 4 m and 8 m from the origin on X-axis. Locate the position of center of mass of the two point masses (i) from the origin and (ii) from 3 kg mass.

Solution

Let us take, $m_1 = 3$ kg and $m_2 = 5$ kg

(i) To find center of mass from the origin:

The point masses are at positions, $x_1 = 4$ m, $x_2 = 8$ m from the origin along X axis.



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{CM} = \frac{(3 \times 4) + (5 \times 8)}{3 + 5}$$

$$x_{CM} = \frac{12 + 40}{8} = \frac{52}{8} = 6.5\,\text{m}$$

The position vectors of two point masses 10 kg and 5 kg are $(-3\hat{i}+2\hat{j}+4\hat{k})$ m and $(3\hat{i}+6\hat{j}+5\hat{k})$ m respectively. Locate the position of center of mass.

$$m_1 = 10\,kg$$

$$m_2 = 5\,kg$$

$$\vec{r}_1 = (-3\hat{i} + 2\hat{j} + 4\hat{k})\,m$$

$$\vec{r}_2 = (3\hat{i} + 6\hat{j} + 5\hat{k})\,m$$

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\therefore \vec{r} = \frac{10(-3\hat{i} + 2\hat{j} + 4\hat{k}) + 5(3\hat{i} + 6\hat{j} + 5\hat{k})}{10+5}$$

$$= \frac{-30\hat{i} + 20\hat{j} + 40\hat{k} + 15\hat{i} + 30\hat{j} + 25\hat{k}}{15}$$

$$= \frac{-15\hat{i} + 50\hat{j} + 65\hat{k}}{15}$$

$$\vec{r} = \left(-\hat{i} + \frac{10}{3}\hat{j} + \frac{13}{3}\hat{k}\right)\,m$$

The center of mass is located at position $\vec{r}$.

CLASS WORK 1

No 1

Two bodies of 2 kg & 4 kg are moving with velocities 20 m/s and 10 m/s respectively towards each other under mutual gravitational attraction. Find the velocity of their centre of mass in m/s.

A. 0 **B. 6** **C. 8** **D. 2** **E. 1**

SOLTION TO CLASS WORK 1

Given:

m_1 = 2 kg, m_2 = 4 kg, v_1 = 20 m/s, v_2 = -10 m/s (negative because direction is opposite)

Applying,

$$V_{cm} = \frac{m_1v_1+m_2v_2}{m_1+m_2}$$

$$V_{cm} = \frac{(2)(20)+(4)(-10)}{2+4}$$

$\mathbf{V_{cm} = 0\ m/s}$

CLASS WORK 2

Two objects of mass 10 kg and 20 kg respectively are connected to the two ends of a rigid rod of length 10 m with negligible mass. The distance of the center of mass of the system from the 10 kg mass is:

A. 5 m **B. 10/3 m** **C. 20/3 m** **D. 10** **E. 2**

SOLTION TO CLASS WORK 2

Let a system of two particles of masses M_1 and M_2 located at points A and B respectively. Let X_1 and X_2 be the position of the particles relative to a fixed origin O. Then, the position X of the center of mass of the system can be calculated using the formula:

$$Centre\ of\ mass,\ X = \frac{M_1X_1 + M_2X_2}{M_1 + M_2}$$

Given,

M_1 = 10 kg, M_2 = 20 kg, Length of rod = 10 m

Let M_1 is at origin, then X_1 and X_2 is 0 and 10 m respectively.

$$Centre\ of\ mass,\ X = \frac{M_1X_1 + M_2X_2}{M_1 + M_2}$$

Putting the values in above equation we get,

$$X = \frac{10\times0+20\times10}{10+20} = \frac{200}{30} = \frac{20}{3}$$

Distance of the center of mass of the system from the 10 kg mass is: **20/3 m.**